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The production approach to markup estimation often measures input distortions ☆

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ABSTRACT

The production approach recovers markups using the output elasticity for a variable and undistorted input. We show using the revenue elasticity for a variable input recovers that input's wedge. Our result has two implications. First, in the canonical setting with CES demand and monopolistic competition, past research using the production approach with revenue data should be recast as evidence of input, rather than output, distortions. Second, future research can use the production approach with revenue data to study input distortions, provided researchers can measure inputs in physical units. A promising application pertains to labor market distortions.

1. Introduction

The production approach to markup estimation is widely used to study output market power. Without requiring a stand on the market structure or demand system, it characterizes markups in a variety of economic models, including those of superstar firms (Autor et al., 2020), trade liberalization (Edmond et al., 2015), production networks (Baqaee and Farhi, 2019), and endogenous growth (Peters, 2020).

Pioneered by Hall (1988) and De Loecker and Warzynski (2012), the production approach recovers firm i's markup μ_i using the ratio estimand given by the right-hand side of Eq. (1).

$$\mu_i = \frac{L_i}{Q_i}\frac{\partial Q_i}{\partial L_i}\left(\frac{W_i L_i}{R_i}\right)^{-1} \tag{1}$$

This estimand is the ratio of the output elasticity for a variable and undistorted input L_i, measured in physical units with input price W_i, to that input's expenditure share of revenue $R_i := P_i Q_i$, where P_i and Q_i denote output price and quantity.

The output elasticity $(L_i/Q_i)\,\partial Q_i/\partial L_i$ measures the percent increase in output quantity Q_i to a one percent increase in input quantity L_i. To estimate the output elasticity, one requires micro data on output and input quantities. But most production datasets only contain information on firm revenues, rather than separating output prices and quantities. This data constraint motivates the study of revenue elasticities. The revenue elasticity $(L_i/R_i)\,\partial R_i/\partial L_i$ measures the percent increase in revenue R_i to a one percent increase in input quantity L_i.

Building on Klette and Griliches (1996), Bond et al. (2021) show profit maximization implies the ratio estimand using the revenue elasticity for a variable and undistorted input is identically equal to one, and hence does not recover the markup.

$$1 = \frac{L_i}{R_i}\frac{\partial R_i}{\partial L_i}\left(\frac{W_i L_i}{R_i}\right)^{-1} \tag{2}$$

Eq. (2) begs the question: How should we interpret the ratio estimand using the revenue elasticity when it differs from one? Our key result in Eq. (3) provides an answer by showing the ratio estimand using the revenue elasticity for a variable input L_i recovers that input's wedge $\tau_i \geq 0$.

$$1 + \tau_i = \frac{L_i}{R_i}\frac{\partial R_i}{\partial L_i}\left(\frac{W_i L_i}{R_i}\right)^{-1} \tag{3}$$

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If the input L_i is undistorted ($\tau_i = 0$), then our result collapses to Eq. (2).

Our result has two important implications. The first concerns past research. When constrained to revenue data, researchers typically proxy for physical output using revenue deflated with an industry price index. This practice recovers output elasticities only under two special cases: perfect competition or no firm heterogeneity in output prices. Meanwhile, in the canonical setting with CES demand and monopolistic competition (Dixit and Stiglitz, 1977; Krugman, 1979; Melitz, 2003), we show that only revenue elasticities can be recovered from revenue data. Following Eq. (3), we can reinterpret the resulting ratio estimand as evidence of input, rather than output, distortions.

The second implication concerns future research. We show researchers can use the production approach to study input distortions even when constrained to revenue data, provided they can measure inputs in physical units. This provision, either by direct observation or structural modeling, is essential because there is a symmetric omitted input price issue (see De Loecker and Goldberg, 2014; Grieco et al., 2016). An important application pertains to labor market distortions, as most production datasets provide information on labor quantities.

2. Recovering firm wedges using production elasticities

In this section, we assume knowledge of output and revenue elasticities. In Section 4, we discuss the identification of the revenue elasticities required to recover input wedges.

2.1. Recovering markups and input wedges using output elasticities

Firm i produces gross output Q_i using two variable inputs: labor L_i and materials M_i. We assume L_i is distorted, whereas M_i is not. The production technology $Q_i = \mathcal{Q}_i(L_i, M_i)$ is twice continuously differentiable and strictly increasing and strictly concave in both arguments. The output elasticities are

$$\theta_{L,i} := \frac{L_i}{Q_i}\frac{\partial \mathcal{Q}_i(\cdot)}{\partial L_i},$$
$$\theta_{M,i} := \frac{M_i}{Q_i}\frac{\partial \mathcal{Q}_i(\cdot)}{\partial M_i}.$$

Let τ_i denote the reduced form labor wedge, which can reflect several underlying distortions.[1] Let W_i and V_i denote the unit input prices of L_i and M_i, respectively. Taking $\{Q_i, \tau_i, W_i, V_i\}$ as given, firm i chooses $\{L_i, M_i\}$ to minimize its total variable cost, subject to its technology constraint (with Lagrange multiplier λ_i).

$$\begin{aligned} \mathcal{C}_i(Q_i; \tau_i, W_i, V_i) \; &:= \; \min_{L_i, M_i} \{(1+\tau_i) W_i L_i + V_i M_i\} \\ &\text{s.t.} \;\; Q_i = \mathcal{Q}_i(L_i, M_i) \end{aligned}$$

By the envelope theorem, λ_i is equal to marginal cost $\partial \mathcal{C}_i(\cdot)/\partial Q_i$. We define the firm's markup μ_i as the ratio of the output price P_i to marginal cost λ_i.

As shown by Hall (1988) and De Loecker and Warzynski (2012), the first-order condition with respect to the undistorted input M_i recovers the markup from the ratio of the output elasticity $\theta_{M,i}$ to the expenditure share $\alpha_{M,i} := V_i M_i / R_i$.

$$\frac{\theta_{M,i}}{\alpha_{M,i}} = \mu_i \tag{4}$$

Meanwhile, the first-order condition with respect to the distorted input L_i yields

$$\frac{\theta_{L,i}}{\alpha_{L,i}} = \mu_i(1+\tau_i) \tag{5}$$

where $\alpha_{L,i} := W_i L_i / R_i$ is the expenditure share of L_i.

The ratio estimand (5) confounds the markup μ_i with the input wedge $\tau_{L,i}$. There are at least two approaches to identify markups and input wedges separately. First, one can impose structure on input or output markets and introduce non-production data to estimate this structural model. Rubens (2021) pursues this strategy by using a discrete choice model of input supply. The drawback of this first approach is that additional structure and data limit focus to particular economic settings.

The second approach uses the "double-ratio estimand" (Dobbelaere and Mairesse, 2013; Morlacco, 2020).

$$\frac{\theta_{L,i}}{\alpha_{L,i}}\left(\frac{\theta_{M,i}}{\alpha_{M,i}}\right)^{-1} = 1+\tau_i \tag{6}$$

Intuitively, both the markup μ_i and the input wedge τ_i depress demand for inputs. Markups lower input demand in proportion to output elasticities. The input wedge τ_i additionally lowers the demand for the distorted input L_i. The double ratio estimand (6) nets out the common decrease in demand from the markup, recovering the input wedge. However, the drawback of this second approach is that knowledge of output elasticities $(\theta_{L,i}, \theta_{M,i})$ is required. It is challenging to identify output elasticities when revenue is the measure of output.[2] For this reason, we propose an alternative method for recovering input wedges that does not require knowledge of output elasticities.

2.2. Recovering input wedges using revenue elasticities

Let $\mathcal{R}_i(L_i, M_i)$ denote firm i's reduced form revenue function.[3] The revenue elasticities are

$$\gamma_{L,i} := \frac{L_i}{R_i}\frac{\partial \mathcal{R}_i(\cdot)}{\partial L_i} = \frac{\theta_{L,i}}{\mu_i},$$
$$\gamma_{M,i} := \frac{M_i}{R_i}\frac{\partial \mathcal{R}_i(\cdot)}{\partial M_i} = \frac{\theta_{M,i}}{\mu_i}.$$

With output market power ($\mu_i > 1$), each revenue elasticity is strictly less than the corresponding output elasticity. Taking $\{\tau_i, W_i, V_i\}$ as given, the firm's static profit maximization problem is

$$\Pi_i(\tau_i, W_i, V_i) := \max_{L_i, M_i} \{\mathcal{R}_i(L_i, M_i) - (1+\tau_i) W_i L_i - V_i M_i\}.$$

The first-order conditions imply

$$\frac{\gamma_{M,i}}{\alpha_{M,i}} = 1, \tag{7}$$
$$\frac{\gamma_{L,i}}{\alpha_{L,i}} = 1+\tau_i. \tag{8}$$

As shown by Bond et al. (2021), the ratio estimand (7) using the revenue elasticity $\gamma_{M,i}$ of the undistorted input M_i is identically equal to one, for all values of the true underlying μ_i. Our new result in Eq. (8) shows the ratio estimand using the revenue elasticity $\gamma_{L,i}$ of the distorted input L_i recovers that input's wedge.

If researchers wish to recover input wedges, but are constrained to only knowing revenue elasticities, rather than output elasticities, then the ratio estimand (8) presents a feasible alternative to the double ratio estimand (6).

[1] We provide a particular microfoundation based on labor market power in Appendix A.1.

[2] See Klette and Griliches (1996), Bond et al. (2021), Kirov and Traina (2021).

[3] The revenue function $\mathcal{R}_i(L_i, M_i)$ implicitly depends on the underlying production technology, demand system, and market structure. Our method for recovering the firm's input wedges using the revenue elasticity does not depend on these primitives. However, the identification of revenue elasticities from production data requires taking a stand on these primitives, as we detail in Section 4.

3. Reinterpreting past findings

We discuss limitations applying to past research attempting to identify output elasticities from revenue data.

3.1. Difficulties in recovering output elasticities from revenue data

Let t index time periods. Without loss of generality, suppose the production technology is Cobb–Douglas, with a Hicks-neutral productivity shock ω_{it}.[4]

$$Q_{it} = \exp(\omega_{it}) L_{it}^{\theta_L} M_{it}^{\theta_M}, \quad \theta_L, \theta_M \in (0, 1) \tag{9}$$

When revenue R_{it} is the measure of physical output, researchers typically use an industry price index P_t to recover deflated revenue $\bar{R}_{it} := R_{it}/P_t$. Given the Cobb–Douglas technology (9), the estimating revenue-production function is

$$\bar{r}_{it} = x_{it}'\theta + \omega_{it} + (p_{it} - p_t)$$

where lower-case variables denote logarithms of the variables in levels, $x_{it} := (l_{it}, m_{it})'$, and the output elasticity vector $\theta := (\theta_L, \theta_M)'$ is the parameter of interest. Given panel data $\{\bar{r}_{it}, x_{it}\}_{i=1,t=1}^{N,T}$, Klette and Griliches (1996) show the OLS estimator of θ is inconsistent due to two sources of bias: (i) the transmission bias arising from unobserved productivity ω_{it} and (ii) the omitted price bias arising from unobserved output prices p_{it}.

$$\begin{aligned}\hat{\theta}_{\text{OLS}} &:= \left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}x_{it}'\right)^{-1}\left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}\bar{r}_{it}\right)\\ &\xrightarrow{p} \theta + \underbrace{\mathbb{E}\left[x_{it}x_{it}'\right]^{-1}\mathbb{E}\left[x_{it}\omega_{it}\right]}_{\text{Transmission Bias}} + \underbrace{\mathbb{E}\left[x_{it}x_{it}'\right]^{-1}\mathbb{E}\left[x_{it}\left(p_{it} - p_t\right)\right]}_{\text{Omitted Price Bias}}\end{aligned}$$

We abstract from the transmission bias and focus on the omitted price bias term. The OLS estimator is consistent if one of either two conditions is true.

1. Perfect competition: This ensures firms can influence neither their own price p_{it} nor the price index p_t, so that $\mathbb{E}[x_{it}(p_{it} - p_t)] = 0$.
2. No firm heterogeneity in output prices: This ensures $p_{it} = p_t, \forall i$ such that p_t fully controls for p_{it}.

CES demand and monopolistic competition. Meanwhile, suppose competition is monopolistic with CES demand, as in canonical models of international trade (Krugman, 1979; Melitz, 2003). The isoelastic inverse demand function is

$$P_{it} = \exp(\xi_{it}) Q_{it}^{-\frac{1}{\eta}}, \quad \eta > 1 \tag{10}$$

where ξ_{it} denotes an idiosyncratic demand shock.

With Cobb–Douglas technology (9) and CES demand (10), the resulting revenue function is log-linear in input quantities.

$$r_{it} = \gamma_L l_{it} + \gamma_M m_{it} + \mu^{-1}\omega_{it} + \xi_{it} \tag{11}$$

The constant revenue elasticities are $\gamma_L = \theta_L/\mu$ and $\gamma_M = \theta_M/\mu$, where $\mu := \eta/(\eta - 1)$ denotes the constant markup. Given a suitable strategy to account for unobserved productivity and demand shocks (ω_{it}, ξ_{it}), the regression Eq. (11) identifies revenue elasticities (γ_L, γ_M) instead of output elasticities (θ_L, θ_M). Given our key result (3), we can reinterpret the resulting ratio estimand as recovering input wedges, instead of the markup.

Proxy methods. Most researchers implementing the production approach with revenue data rely on the two-stage proxy method of Ackerberg et al. (2015) (henceforth ACF). For example, De Loecker and Warzynski (2012) follow ACF in estimating a value-added production function in labor and capital, using the input demand equation for materials to obtain a control function for productivity.

We show the ACF control function approach does not correct for the omitted price bias if it exists. To see why, ACF emphasize that the microfoundation for a value added production function is that gross output Q_{it} is a Leontief function of value added Y_{it} and materials M_{it}. For simplicity, suppose value-added is a Cobb–Douglas function: $Y_{it} = \exp(\omega_{it}) L_{it}^{\theta_L} K_{it}^{\theta_K}$. Then, the gross output function is

$$Q_{it} = \min\left\{\exp(\omega_{it}) L_{it}^{\theta_L} K_{it}^{\theta_K}, \theta_M M_{it}\right\}.$$

Cost-minimization implies the proportion of materials to value-added output is fixed, yielding the input demand equation for M_{it}.

$$\exp(\omega_{it}) L_{it}^{\theta_L} K_{it}^{\theta_K} = \theta_M M_{it} \tag{12}$$

Conditional on (L_{it}, K_{it}), Eq. (12) makes explicit that the proxy variable M_{it} is only informative about ω_{it} and not informative about p_{it}. In other words, the resulting control function for ω_{it} does not correct for the omitted price bias if it exists.

3.2. Reinterpreting two past findings

De Loecker and Warzynski (2012). De Loecker and Warzynski (2012) (henceforth DLW) study the relationship between markups and export status, finding exporting firms charge higher markups on average than domestic producers. In using deflated revenue as their measure of output, DLW are subject to the limitations outlined in Section 3.1. To the extent that these limitations apply, we reinterpret DLW's findings as evidence that labor wedges are higher for exporting firms than domestic producers. Our reinterpretation is consistent with Helpman et al. (2010), who introduce search and matching frictions into the Melitz (2003) framework with heterogeneous firms and predict strictly greater wage inequality in the open economy when only some firms export than in the closed economy.

Raval (2020). Raval (2020) finds the markup distribution is significantly more dispersed when using labor instead of materials as the flexible input. In using deflated revenue as his measure of output, Raval is also subject to the limitations outlined in Section 3.1. To the extent that these limitations apply, we reinterpret his finding as evidence of greater dispersion in the labor wedge than in the materials wedge. Our reinterpretation is consistent with Asker et al. (2014), who find empirically that dispersion in the marginal revenue product of dynamically variable inputs, such as labor, is greater than that of non-dynamically variable inputs, such as materials.

4. Directions for future research

In closing, we provide direction for identifying revenue elasticities in the special case with a Cobb–Douglas technology and CES demand.

Data. We require data on input quantities for labor l_{it} (e.g. employment or hours worked) and materials m_{it} (e.g. kilowatt of energy). We also require data on revenue $\tilde{r}_{it} := r_{it} + \varepsilon_{it}$, which is measured with an error ε_{it} that is mean zero, independent across (i, t), and uncorrelated with $(l_{it}, m_{it}, w_{it}, v_{it})$.[5] Our sample is $\mathcal{S} = \left\{\tilde{r}_{it}, l_{it}, m_{it}\right\}_{i=1,t=1}^{N,T}$, where N is large and T is small.

[4] We do not impose the constant returns to scale restriction $\theta_L + \theta_M = 1$.

[5] Note that we do not require deflated revenue.

Identification. Given a Cobb–Douglas technology (9) and CES demand (10), the estimating regression equation is

$$\tilde{r}_{it} = \gamma_L l_{it} + \gamma_M m_{it} + u_{it} \tag{13}$$

where $u_{it} := \left(\mu^{-1}\omega_{it} + \xi_{it} + \varepsilon_{it}\right)$ is the composite error term. We identify the revenue elasticity vector $\boldsymbol{\gamma} := (\gamma_L, \gamma_M)'$ using the dynamic panel method of Blundell and Bond (2000).

We show in Appendix A.2 that the firm's optimal input demand functions are

$$\begin{aligned} l_{it} &= \ln C_L + \left(\frac{1}{\mu - \theta_L - \theta_M}\right)[\omega_{it} + \mu\xi_{it} - \theta_M v_{it}] \\ &\quad - \left(\frac{\theta_M}{\theta_L + \theta_M} + \frac{\theta_L}{\theta_L + \theta_M}\frac{\mu}{\mu - \theta_L - \theta_M}\right)[\ln(1+\tau_{it}) + w_{it}], \quad (14) \\ m_{it} &= \ln C_M + \left(\frac{1}{\mu - \theta_L - \theta_M}\right)[\omega_{it} + \mu\xi_{it} - \theta_L[\ln(1+\tau_{it}) + w_{it}]] \\ &\quad - \left(\frac{\theta_L}{\theta_L + \theta_M} + \frac{\theta_M}{\theta_L + \theta_M}\frac{\mu}{\mu - \theta_L - \theta_M}\right) v_{it}, \quad (15) \end{aligned}$$

where $\{C_L, C_M\}$ are constants.

There is an endogeneity bias in the OLS estimation of (13). Optimizing behavior in (14) and (15) implies (l_{it}, m_{it}) depend on unobserved productivity and demand shocks (ω_{it}, ξ_{it}), so (l_{it}, m_{it}) are endogenous. Suppose (ω_{it}, ξ_{it}) are serially uncorrelated over time and there is persistent variation across firms in $(\tau_{it}, w_{it}, v_{it})$. Then, $\boldsymbol{\gamma}$ is identified from the moment condition of Blundell and Bond (2000).

$$\mathbb{E}\left[\left(\tilde{r}_{it} - \gamma_L l_{it} - \gamma_M m_{it}\right)\begin{pmatrix} l_{i,t-1} \\ m_{i,t-1}\end{pmatrix}\right] = 0 \tag{16}$$

Intuitively, persistent variation across firms in $(\tau_{it}, w_{it}, v_{it})$ ensures the instruments $\left(l_{i,t-1}, m_{i,t-1}\right)$ are correlated with the endogenous variables (l_{it}, m_{it}), thereby satisfying the rank condition. The role of the labor wedge τ_{it} in providing identifying variation is analogous to that of adjustment costs in Bond and Söderbom (2005). Moreover, since (ω_{it}, ξ_{it}) are serially uncorrelated, $\left(l_{i,t-1}, m_{i,t-1}\right)$ are also orthogonal to (ω_{it}, ξ_{it}), thereby satisfying the exclusion restriction. The moment conditions (16) extend to cases in which (ω_{it}, ξ_{it}) follow low order ARMA processes (see Bond et al., 2021).

5. Conclusion

We show the ratio estimand using the revenue elasticity for a variable input recovers that input's wedge. Under CES demand and monopolistic competition, our result warrants reinterpreting past findings using revenue data as evidence of input, rather than output, distortions. Our result also presents scope for studying input distortions in the absence of output price data, provided researchers can measure inputs in physical units.

Appendix

A.1. Microfoundations for the input wedge

We provide a microfoundation for the labor wedge based on labor market power, under which firm i can influence its wage schedule $\mathcal{W}_i(L_i)$. Abstracting from materials M_i, the profit maximization problem is

$$\max_{L_i}\{\mathcal{R}_i(L_i) - \mathcal{W}_i(L_i)L_i\}.$$

The first-order condition is

$$\frac{\partial \mathcal{R}_i}{\partial L_i} = (1+\nu_i)\mathcal{W}_i(L_i)$$

where the perceived inverse elasticity of labor supply is

$$\nu_i := \frac{\mathcal{W}_i'(L_i)L_i}{\mathcal{W}_i(L_i)} \geq 0.$$

The markdown of the wage relative to the marginal revenue product of labor is

$$\mathcal{W}_i(L_i)\left(\frac{\partial \mathcal{R}_i}{\partial L_i}\right)^{-1} = (1+\nu_i)^{-1} \leq 1.$$

The inequality is strict if the firm has labor market power, *i.e.* $\nu_i > 0$. The ratio estimand using the revenue elasticity of labor recovers this markdown.

$$\frac{\gamma_{Li}}{\alpha_{L,i}} = 1 + \nu_i \geq 1$$

Input market power is just one of many possible microfoundations for input distortions. Others include adjustment costs, financial frictions, and inputs used to influence the demand system. Importantly, multiple underlying distortions map into the same input wedge (Chari et al., 2007). Thus, our method recovers the overall input wedge induced by multiple underlying distortions, rather than separately recovering each one.

Indeed, suppose labor L_i is subject to both input market power and a convex adjustment cost function $\Psi_i(L_i)$, which satisfies $\Psi_i\left(\bar{L}\right) = 0$ and $\Psi_i'\left(\bar{L}\right) = 0$ for some baseline quantity $\bar{L}$. The two relevant labor distortion elasticities are

$$\begin{aligned} \nu_i &:= \frac{\mathcal{W}_i'(L_i)L_i}{\mathcal{W}_i(L_i)} \geq 0, \\ \kappa_i &:= \frac{\Psi'(L_i)L_i}{\Psi(L_i)} - 1 \geq 0. \end{aligned}$$

In the absence of labor distortions, we have $\nu_i = \kappa_i = 0$. The profit maximization problem is

$$\max_{L_i}\{\mathcal{R}_i(L_i) - \mathcal{W}_i(L_i)L_i - \Psi_i(L_i)\}.$$

The first-order condition implies the ratio estimand using the revenue elasticity of labor recovers a cost weighted average of the structural elasticities (ν_i, κ_i)

$$\frac{\gamma_{Li}}{\alpha_{L,i}} = 1 + \frac{\mathcal{W}_i(L_i)L_i}{\mathcal{C}_i(L_i)}\nu_i + \frac{\Psi_i(L_i)}{\mathcal{C}_i(L_i)}\kappa_i$$

where total cost is $\mathcal{C}_i(L_i) := (\mathcal{W}_i(L_i)L_i + \Psi_i(L_i))$. We encourage researchers to rely on the institutional details of their application to justify which underlying input distortion is most significant.

A.2. Optimal input demand functions

Cost minimization. Taking $\{Q_{it}, \omega_{it}, \tau_{it}, W_{it}, V_{it}\}$ as given, the firm chooses $\{L_{it}, M_{it}\}$ to minimize total variable cost, subject to the technology constraint (9) (with Lagrange multiplier λ_{it}).

$$\begin{aligned} \mathcal{C}(Q_{it}; \omega_{it}, \tau_{it}, W_{it}, V_{it}) &:= \min_{L_{it}, M_{it}}\{(1+\tau_{it})W_{it}L_{it} + V_{it}M_{it}\} \\ &\quad \text{s.t.} \quad Q_{it} = \exp(\omega_{it})L_{it}^{\theta_L}M_{it}^{\theta_M} \end{aligned}$$

The first-order conditions are

$$\begin{aligned} (1+\tau_{it})W_{it} &= \theta_L\lambda_{it}\exp(\omega_{it})L_{it}^{\theta_L - 1}M_{it}^{\theta_M}, \\ V_{it} &= \theta_M\lambda_{it}\exp(\omega_{it})L_{it}^{\theta_L}M_{it}^{\theta_M - 1}. \end{aligned}$$

Combining the first-order conditions with Eq. (9) yields the optimal input demand functions, for any given Q_{it}.

$$L_{it} = \left[\left(\frac{\theta_L}{\theta_M}\right)^{\theta_M}\exp(-\omega_{it})((1+\tau_{it})W_{it})^{-\theta_M}V_{it}^{\theta_M}\right]^{\frac{1}{\theta_L+\theta_M}} Q_{it}^{\frac{1}{\theta_L+\theta_M}}, \tag{17}$$

$$M_{it} = \left[\left(\frac{\theta_M}{\theta_L} \right)^{\theta_L} \exp\left(-\omega_{it}\right) \left(\left(1+\tau_{it}\right) W_{it} \right)^{\theta_L} V_{it}^{-\theta_L} \right]^{\frac{1}{\theta_L+\theta_M}} Q_{it}^{\frac{1}{\theta_L+\theta_M}}, \quad (18)$$

Thus, the firm's minimized total variable cost function is

$$\begin{aligned} & \mathcal{C}\left(Q_{it}; \omega_{it}, \tau_{it}, W_{it}, V_{it}\right) \\ &= \left(1+\tau_{L,it}\right) W_{it} L_{it} + V_{it} M_{it} \\ &= A \left(\exp\left(-\omega_{it}\right) \left(\left(1+\tau_{it}\right) W_{it} \right)^{\theta_L} V_{it}^{\theta_M} \right)^{\frac{1}{\theta_L+\theta_M}} Q_{it}^{\frac{1}{\theta_L+\theta_M}} \end{aligned}$$

where $A := \left[\left(\frac{\theta_L}{\theta_M}\right)^{\frac{\theta_M}{\theta_L+\theta_M}} + \left(\frac{\theta_M}{\theta_L}\right)^{\frac{\theta_L}{\theta_L+\theta_M}} \right]$ is a constant.

Profit maximization. Taking its cost function $\mathcal{C}\left(Q_{it}; \omega_{it}, \tau_{it}, W_{it}, V_{it}\right)$ as given, the firm chooses Q_{it} to maximize its profits, subject to the demand system constraint (10).

$$\begin{aligned} \Pi\left(\omega_{it}, \xi_{it}, \tau_{it}, W_{it}, V_{it}\right) &:= \max_{Q_{it}} \left\{ P_{it} Q_{it} - \mathcal{C}\left(Q_{it}; \omega_{it}, \tau_{it}, W_{it}, V_{it}\right) \right\} \\ & \text{s.t.} \quad P_{it} = \exp\left(\xi_{it}\right) Q_{it}^{-\frac{1}{\eta}} \end{aligned}$$

The first-order condition yields

$$\begin{aligned} & Q_{it}^{\frac{1}{\theta_L+\theta_M}} \\ &= \left[B \exp\left(\omega_{it}\right)^{\frac{1}{\theta_L+\theta_M}} \exp\left(\xi_{it}\right) \left(\left(\left(1+\tau_{it}\right) W_{it} \right)^{\theta_L} V_{it}^{\theta_M} \right)^{-\frac{1}{\theta_L+\theta_M}} \right]^{\frac{\mu}{\mu-\theta_L-\theta_M}} \end{aligned} \quad (19)$$

where $B := \mu^{-1}\left(\theta_L+\theta_M\right) A^{-1}$ is a constant. We use Eq. (19) to substitute for $Q_{it}^{\frac{1}{\theta_L+\theta_M}}$ in Eqs. (17) and (18) to derive the optimal input demand functions

$$\begin{aligned} L_{it} &= C_L \left[\exp\left(\omega_{it}\right) \exp\left(\xi_{it}\right)^{\mu} V_{it}^{-\theta_M} \right]^{\frac{1}{\mu-\theta_L-\theta_M}} \times \\ & \quad \left[\left(1+\tau_{it}\right) W_{it} \right]^{-\left(\frac{\theta_M}{\theta_L+\theta_M} + \frac{\theta_L}{\theta_L+\theta_M} \frac{\mu}{\mu-\theta_L-\theta_M} \right)}, \\ M_{it} &= C_M \left[\exp\left(\omega_{it}\right) \exp\left(\xi_{it}\right)^{\mu} \left(\left(1+\tau_{it}\right) W_{it} \right)^{-\theta_L} \right]^{\frac{1}{\mu-\theta_L-\theta_M}} \times \\ & \quad V_{it}^{-\left(\frac{\theta_L}{\theta_L+\theta_M} + \frac{\theta_M}{\theta_L+\theta_M} \frac{\mu}{\mu-\theta_L-\theta_M} \right)}, \end{aligned}$$

where $C_L := \left(\frac{\theta_L}{\theta_M}\right)^{\frac{\theta_M}{\theta_L+\theta_M}} B^{\frac{\mu}{\mu-\theta_L-\theta_M}}$ and $C_M := \left(\frac{\theta_M}{\theta_L}\right)^{\frac{\theta_L}{\theta_L+\theta_M}} B^{\frac{\mu}{\mu-\theta_L-\theta_M}}$ are constants.

References

Ackerberg, D.A., Caves, K., Frazer, G., 2015. Identification properties of recent production function estimators. Econometrica 83 (6), 2411–2451. http://dx.doi.org/10.3982/ECTA13408.

Asker, J., Collard-Wexler, A., De Loecker, J., 2014. Dynamic inputs and resource Mis)Allocation. J. Political Econ. 122 (5), 1013–1063. http://dx.doi.org/10.1086/677072.

Autor, D., Dorn, D., Katz, L.F., Patterson, C., Van Reenen, J., 2020. The fall of the labor share and the rise of superstar firms. Q. J. Econ. 135 (2), 645–709. http://dx.doi.org/10.1093/qje/qjaa004.

Baqaee, D.R., Farhi, E., 2019. Productivity and misallocation in general equilibrium. Q. J. Econ. 135 (1), 105–163. http://dx.doi.org/10.1093/qje/qjz030.

Blundell, R., Bond, S., 2000. GMM Estimation with persistent panel data: an application to production functions. Econ. Rev. 19 (3), 321–340. http://dx.doi.org/10.1080/07474930008800475.

Bond, S., Hashemi, A., Kaplan, G., Zoch, P., 2021. Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data. J. Monet. Econ. 121, 1–14. http://dx.doi.org/10.1016/j.jmoneco.2021.05.004.

Bond, S., Söderbom, M., 2005. Adjustment Costs and the Identification of Cobb Douglas Production Functions. (W05/04), Institute for Fiscal Studies, https://ideas.repec.org/p/ifs/ifsewp/05-04.html.

Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2007. Business cycle accounting. Econometrica 75 (3), 781–836. http://dx.doi.org/10.1111/j.1468-0262.2007.00768.x.

De Loecker, J., Goldberg, P.K., 2014. Firm performance in a global market. Ann. Rev. Econ. 6 (1), 201–227. http://dx.doi.org/10.1146/annurev-economics-080113-104741.

De Loecker, J., Warzynski, F., 2012. Markups and firm-level export status. Am. Econ. Rev. 102 (6), 2437–2471. http://dx.doi.org/10.1257/aer.102.6.2437.

Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. Am. Econ. Rev. 67 (3), 297–308, https://ideas.repec.org/a/aea/aecrev/v67y1977i3p297-308.html.

Dobbelaere, S., Mairesse, J., 2013. Panel data estimates of the production function and product and labor market imperfections. J. Appl. Econ. 28 (1), 1–46. http://dx.doi.org/10.1002/jae.1256.

Edmond, C., Midrigan, V., Xu, D.Y., 2015. Competition, markups, and the gains from international trade. Am. Econ. Rev. 105 (10), 3183–3221. http://dx.doi.org/10.1257/aer.20120549.

Grieco, P.L.E., Li, S., Zhang, H., 2016. Production function estimation with unobserved input price dispersion. Int. Econ. Rev. 57 (2), 665–690. http://dx.doi.org/10.1111/iere.12172.

Hall, R.E., 1988. The relation between price and marginal cost in U.S. Industry. J. Political Econ. 96 (5), 921–947. http://dx.doi.org/10.1086/261570.

Helpman, E., Itskhoki, O., Redding, S., 2010. Inequality and unemployment in a global economy. Econometrica 78 (4), 1239–1283. http://dx.doi.org/10.3982/ECTA8640.

Kirov, I., Traina, J., 2021. Measuring Markups with Revenue Data. Tech. rep., http://dx.doi.org/10.2139/ssrn.3912966.

Klette, T.J., Griliches, Z., 1996. The inconsistency of common scale estimators when output prices are unobserved and endogenous. J. Appl. Econ. 11 (4), 343–361. http://dx.doi.org/10.1002/(SICI)1099-1255(199607)11:4<343::AID-JAE404>3.0.CO;2-4.

Krugman, P.R., 1979. Increasing returns, monopolistic competition, and international trade. J. Int. Econ. 9 (4), 469–479. http://dx.doi.org/10.1016/0022-1996(79)90017-5.

Melitz, M.J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71 (6), 1695–1725. http://dx.doi.org/10.1111/1468-0262.00467.

Morlacco, M., 2020. Market Power in Input Markets: Theory and Evidence from French Manufacturing. Tech. rep..

Peters, M., 2020. Heterogeneous markups, growth, and endogenous misallocation. Econometrica 88 (5), 2037–2073. http://dx.doi.org/10.3982/ECTA15565.

Raval, D., 2020. Testing the Production Approach to Markup Estimation. Tech. rep., http://dx.doi.org/10.2139/ssrn.3324849.

Rubens, M., 2021. Market Structure, Oligopsony Power, and Productivity. Tech. rep., http://dx.doi.org/10.2139/ssrn.3800254.